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The design of optimal guidance law with multi-constraints using block pulse functions

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ABSTRACT

When there are time-varying parameters in motion equations, the optimal guidance law with multi-constraints generally cannot be solved analytically. Based on block pulse functions, a design method of optimal guidance law is presented combining optimal control theory and numerical value computation for time-varying systems. The presented guidance law can optimize the combination of landing angle, miss distance, and control energy consumption. Using both the proposed guidance law and the proportional navigation law, ballistic simulations are made. Compared to the proportional navigation law, the optimal guidance law is able to more than double the landing angle. Because of the steep terminal trajectory, the strike accuracy and damage effects are increased. Averaging the time-varying coefficients of the optimal guidance law, a suboptimal guidance law is obtained. This guidance law is simpler and can make the terminal trajectory steep too. Therefore, it could be applied to projects more easily and requires less onboard computational resources. However, it consumes slightly more control energy than the optimal guidance law.

1. Introduction

Global positioning system (GPS)-based guidance systems have fire and forget feature, are weatherproof and low cost per kill, and have therefore received much attention from researchers [13, 1–4,11]. The vertical measurement error of GPS is larger than its horizontal error [9]. In order to reduce its effect on guidance accuracy, a near-vertical descent is required in the terminal guidance phase. The terminal flight-path angle is expected to be close to, but not equal, -90° because the constraints of volume and cost limit the control capability of guided munitions. The large landing angle can also increase the damage effect and penetration capacity of guided munitions. Previous guidance law research related to angle constraints may be largely divided into two categories: optimal [15,10,14,8] and not optimal. The latter is usually more likely to have some advantages, such as having a simpler form, not requiring range-to-target information [12], not requiring a time-to-go t_{go} estimation [7], and being robust [6,5]. To optimize the combination of landing angle, miss distance, and control energy consumption, research is needed regarding the optimal guidance law with multi-constraints.

In order to design the optimal guidance law, motion equations should first be established. These equations often include time-varying parameters. However, for time-varying systems, the optimal guidance law cannot generally be solved analytically. Therefore, time-varying parameters are not intended to appear in the design of guidance law. When time-varying parameters are inevitable, there are two methods that may solve the problem. One method is substituting constants for time-varying parameters. This method is simple, and is therefore employed by many studies; however, the simplification critically decreases the effect of the designed guidance law. In fact, this method cannot be applied when the variation range of the time-varying parameters is large. The other method is combining the optimal control theory and the numerical value method. However, the method has not been actively reported in the design of optimal guidance law with multi-constraints.

Block pulse function series are orthogonal functions with a piecewise constant value. They are simple and easy operations. In this paper, a block pulse function series is applied in order to solve the problem described above. For time-varying control systems, a design method is proposed for the optimal guidance law with multi-constraints combining the numerical value method and the optimal control theory.

2. Motion equations

Consider the homing guidance geometry shown in Fig. 1. Here, M and T denote the guided projectile and target, respectively, and they are regarded as particles whose movements are in a vertical

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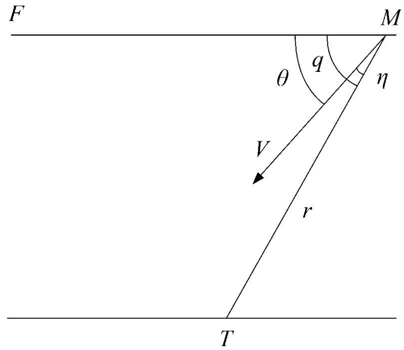

Fig. 1. Homing guidance geometry.

plane. Take the points M, MF, and $r = MT$ as the pole, polar axis, and projectile-target distance, respectively. V is the projectile velocity, and θ is its flight-path angle. GPS-guided munitions are used against stationary targets; thus, the target velocity is zero. The polar equations of motion in this homing problem are given by

$$\dot{r} = -V\cos(q-\theta), \quad \text{and} \tag{1}$$

$$r\dot{q} = V\sin(q-\theta) \tag{2}$$

Differentiating (2) with respect to time and substituting (1) into it, we have

$$\ddot{q} = \left(\frac{\dot{V}}{V} - 2\frac{\dot{r}}{r}\right)\dot{q} + \frac{\dot{r}}{r}\dot{\theta} \tag{3}$$

In general, GPS-guided munitions have little or no thrust. Therefore, they cannot maintain a constant axial velocity, and their velocity changes slowly. Thus, $\dot{V}/V \ll 1$ and $\dot{V}/V = 0$ is assumed. Thus, (3) can be written as

$$\ddot{q} = -2\frac{\dot{r}}{r}\dot{q} + \frac{\dot{r}}{r}\dot{\theta} \tag{4}$$

Let $x_1(t) = q - \theta_F, x_2(t) = \dot{x}_1(t) = \dot{q}$, and $u(t) = \dot{\theta}$. Here, θ_F is the expected flight-path angle in landing moment $t_f, \theta_F = \theta(t_f)$, and $u(t)$ is the guidance law that we want to obtain. Therefore, from (4), we have

$$\dot{x}_1(t) = x_2(t) \tag{5}$$

$$\dot{x}_2(t) = -2\frac{\dot{r}}{r}x_2(t) + \frac{\dot{r}}{r}u(t) \tag{6}$$

Let $g(t) = -\dot{r}/r$. Therefore, from (6), we have

$$\dot{x}_2(t) = 2g(t)x_2(t) - g(t)u(t) \tag{7}$$

Based on (5) and (7), the motion equations can be written as

$$\dot{\boldsymbol{X}}(t) = A(t)\boldsymbol{X}(t) + \boldsymbol{B}(t)u(t) \tag{8}$$

where

$$\boldsymbol{X}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \qquad \boldsymbol{A}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 2g(t) \end{bmatrix}, \qquad \boldsymbol{B}(t) = \begin{bmatrix} 0 \\ -g(t) \end{bmatrix}$$

Initial conditions: when $t = t_0$, $x_1(t_0) = q(t_0) - \theta_F$, and $x_2(t_0) = \dot{q}(t_0)$.

3. Optimal guidance law design on the basis of block pulse functions

In the guided process, the control energy consumption should be as small as possible, the terminal projectile should be as steep as possible, and the landing angle should be close to 90°. Therefore, the performance index can be defined as follows:

$$J = \boldsymbol{X}^{\mathrm{T}}(t_f)\boldsymbol{C}\boldsymbol{X}(t_f) + \frac{1}{2}\int_{t_0}^{t_f} R(t)u^2(t)\,\mathrm{d}t \tag{9}$$

where $\boldsymbol{C} = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $R(t) = 1$, and $t_f \cong t_0 + \frac{1}{g(t_0)}$. Based on the optimal control theory [17], the optimal guidance law for the system above is

$$u(t) = \boldsymbol{K}(t)\boldsymbol{X}(t) \tag{10}$$

where

$$\boldsymbol{K}(t) = -R^{-1}\boldsymbol{B}^{\mathrm{T}}(t)\boldsymbol{P}(t) \tag{11}$$

Here, $\boldsymbol{K}(t)$ is the guidance law coefficient matrix that will be obtained by the following solution procedure and $\boldsymbol{P}$ satisfies the Riccati matrix differential equation:

$$\begin{aligned} \dot{\boldsymbol{P}}(t) = {} & -\boldsymbol{P}(t)\boldsymbol{A}(t) - \boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{P}(t) \\ & + \boldsymbol{P}(t)\boldsymbol{B}(t)R^{-1}(t)\boldsymbol{B}^{\mathrm{T}}(t)\boldsymbol{P}(t) - \boldsymbol{Q}(t) \end{aligned} \tag{12}$$

where $\boldsymbol{Q}(t) = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right]$, and the terminal condition of $\boldsymbol{P}(t)$ is

$$\boldsymbol{P}(t_f) = \boldsymbol{C} \tag{13}$$

Eq. (12) can be decomposed into the following two matrix differential equations:

$$\dot{\boldsymbol{W}}(t) = \boldsymbol{A}(t)\boldsymbol{W}(t) - \boldsymbol{B}(t)\boldsymbol{R}^{-1}(t)\boldsymbol{B}^{\mathrm{T}}(t)\boldsymbol{Y}(t) \tag{14}$$

$$\dot{\boldsymbol{Y}}(t) = -\boldsymbol{Q}(t)\boldsymbol{W}(t) - \boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{Y}(t) \tag{15}$$

The terminal conditions of $\boldsymbol{W}(t)$ and $\boldsymbol{Y}(t)$ are

$$\boldsymbol{W}(t_f) = \boldsymbol{I} \tag{16}$$

$$\boldsymbol{Y}(t_f) = \boldsymbol{C} \tag{17}$$

where $\boldsymbol{I}$ is the second order unit matrix. The solution for the Riccati equation is:

$$\boldsymbol{P}(t) = \boldsymbol{Y}(t)\boldsymbol{W}^{-1}(t) \tag{18}$$

Rearrange (14) and (15) as follows:

$$\dot{\boldsymbol{Z}}(t) = \boldsymbol{F}(t)\boldsymbol{Z}(t) \tag{19}$$

where

$$\boldsymbol{Z}(t) = \begin{bmatrix} \boldsymbol{W}(t) \\ \boldsymbol{Y}(t) \end{bmatrix} \tag{20}$$

$$\boldsymbol{F}(t) = \begin{bmatrix} \boldsymbol{A}(t) & -\boldsymbol{B}(t)R^{-1}(t)\boldsymbol{B}^{\mathrm{T}}(t) \\ -\boldsymbol{Q}(t) & -\boldsymbol{A}^{\mathrm{T}}(t) \end{bmatrix} \tag{21}$$

Integrating (19) from t_f to t, we can derive the following:

$$\boldsymbol{Z}(t) - \boldsymbol{Z}(t_f) = \int_{t_f}^{t} \boldsymbol{F}(\tau)\boldsymbol{Z}(\tau)\,\mathrm{d}\tau \tag{22}$$

Expanding (22) into block pulse series, we have

$$\begin{aligned} & [\,\boldsymbol{Z}_1 \quad \boldsymbol{Z}_2 \quad \cdots \quad \boldsymbol{Z}_m\,]\boldsymbol{\Phi}(t) - [\,\boldsymbol{Z}(t_f) \quad \boldsymbol{Z}(t_f) \quad \cdots \quad \boldsymbol{Z}(t_f)\,]\boldsymbol{\Phi}(t) \\ & \quad \cong -[\,\boldsymbol{F}_1\boldsymbol{Z}_1 \quad \boldsymbol{F}_2\boldsymbol{Z}_2 \quad \cdots \quad \boldsymbol{F}_m\boldsymbol{Z}_m\,]h\boldsymbol{H}^{\mathrm{T}}\boldsymbol{\Phi}(t) \end{aligned} \tag{23}$$

where

$$\boldsymbol{H}=\frac{1}{2}\begin{bmatrix}1 & 2 & 2 & \cdots & 2\\ 0 & 1 & 2 & \cdots & 2\\ 0 & 0 & 1 & \cdots & 2\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\end{bmatrix},\quad \text{and} \tag{24}$$

$$\boldsymbol{\Phi}(t)=[\,\phi_1(t)\quad \phi_2(t)\quad \cdots\quad \phi_m(t)\,]^{\mathrm{T}} \tag{25}$$

Here, $\phi_i(t)$ is a block pulse function, $i=1,2,\ldots,m$; m is the number of block pulse functions, and h is the block pulse width. $\boldsymbol{F}_k$ is the mean value of $\boldsymbol{F}(t)$ in the interval $[(k-1)T/m, kT/m]$. The definition and some elementary properties of block pulse functions can be found in Appendix A. From (23), we have

$$\boldsymbol{Z}_1-\boldsymbol{Z}(t_f)\cong-\frac{1}{2}h(\boldsymbol{F}_1\boldsymbol{Z}_1+2\boldsymbol{F}_2\boldsymbol{Z}_2+\cdots+2\boldsymbol{F}_m\boldsymbol{Z}_m) \tag{26}$$

$$\boldsymbol{Z}_2-\boldsymbol{Z}(t_f)\cong-\frac{1}{2}h(\boldsymbol{F}_2\boldsymbol{Z}_2+2\boldsymbol{F}_3\boldsymbol{Z}_3+\cdots+2\boldsymbol{F}_m\boldsymbol{Z}_m) \tag{27}$$

$$\vdots$$

$$\boldsymbol{Z}_{m-1}-\boldsymbol{Z}(t_f)\cong-\frac{1}{2}h(\boldsymbol{F}_{m-1}\boldsymbol{Z}_{m-1}+2\boldsymbol{F}_m\boldsymbol{Z}_m) \tag{28}$$

$$\boldsymbol{Z}_m-\boldsymbol{Z}(t_f)\cong-\frac{1}{2}h(\boldsymbol{F}_m\boldsymbol{Z}_m) \tag{29}$$

Solving Eqs. (26)–(29), we obtain the following recursive algorithm:

$$\boldsymbol{Z}_m\cong\left(\boldsymbol{I}+\frac{1}{2}h\boldsymbol{F}_m\right)^{-1}\boldsymbol{Z}(t_f) \tag{30}$$

$$\boldsymbol{Z}_{k-1}\cong\left(\boldsymbol{I}+\frac{1}{2}h\boldsymbol{F}_{k-1}\right)^{-1}\left(\boldsymbol{I}-\frac{1}{2}h\boldsymbol{F}_k\right)\boldsymbol{Z}_k,\quad k=2,3,\ldots,m \tag{31}$$

From (20), (30), and (31), we can obtain $\boldsymbol{W}_k(t)$ and $\boldsymbol{Y}_k(t)$, $k=1,2,\ldots,m$. Eq. (11) can be written as

$$\begin{aligned}\boldsymbol{K}(t)&\cong[\,\boldsymbol{K}_1\quad \boldsymbol{K}_2\quad \cdots\quad \boldsymbol{K}_m\,]\boldsymbol{\Phi}(t)\\ &=-\left[\sum_{k=1}^{m}R_k^{-1}\phi_k(t)\right]\cdot\left[\sum_{l=1}^{m}\boldsymbol{B}_l^{\mathrm{T}}\phi_l(t)\right]\\ &\quad\cdot\left[\sum_{i=1}^{m}\boldsymbol{Y}_i\phi_i(t)\right]\cdot\left[\sum_{j=1}^{m}\boldsymbol{W}_j^{-1}\phi_j(t)\right]\end{aligned} \tag{32}$$

where

$$\boldsymbol{K}_i=[\,k_{1,i}\quad k_{2,i}\,]^{\mathrm{T}},\quad i=1,2,\ldots,m \tag{33}$$

R_k is the mean value of $R(t)$ in the interval $[(k-1)T/m, kT/m]$, and $\boldsymbol{B}_l$ is the mean value of $\boldsymbol{B}(t)$ in the interval $[(l-1)T/m, lT/m]$. Based on the orthogonality, (32) can be written as

$$\boldsymbol{K}(t)\cong-\sum_{k=1}^{m}R_k^{-1}\boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{Y}_k\boldsymbol{W}_k^{-1}\phi_k(t) \tag{34}$$

4. Simulation results

In a ground coordinate system, let the muzzle velocity of the guided projectile be 800 m/s, and its initial flight-path angle θ_0 is 30°. The muzzle coordinates are $(0,0)$, and the target coordinates are $(20\,000,0)$. The guidance will begin when the projectile reaches the highest point. Figs. 2 and 3 show the histories of k_1 and k_2 versus t_{go}, respectively, where $t_{go}=t_f-t$. Ballistic simulations were conducted using three kinds of guidance laws, and the results were compared. The first guidance law is from the proposed method, called the optimal guidance law. The second is the suboptimal guidance law, which is obtained by taking the means of k_1 and k_2 as guidance law coefficients. The third is the proportional navigation law.

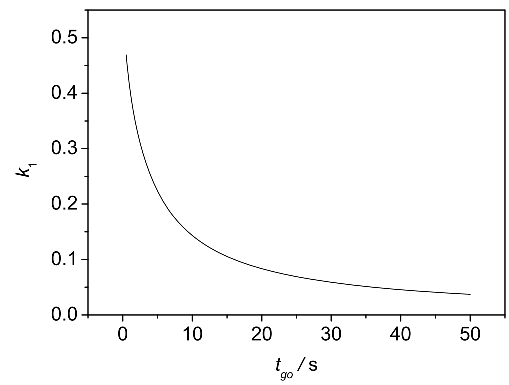

Fig. 2. k_1 history vs. t_{go}.

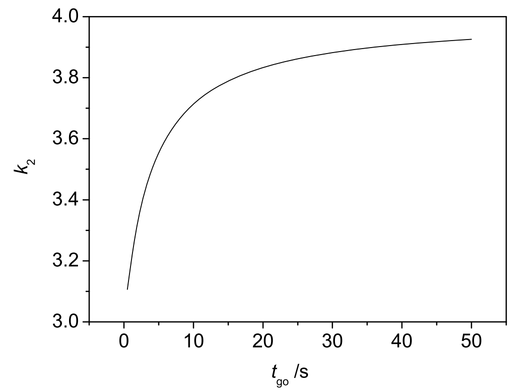

Fig. 3. k_2 history vs. t_{go}.

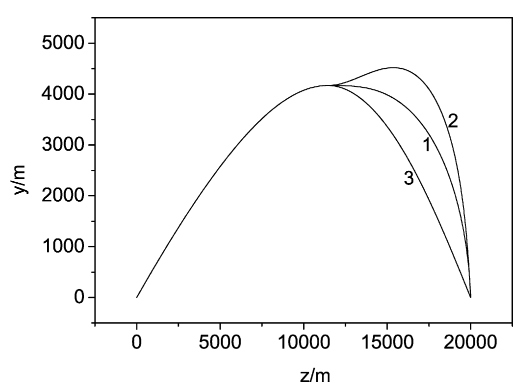

Fig. 4. Trajectories using three different guidance laws, respectively.

Fig. 4 compares the three trajectories achieved with the three guidance laws above. Lines 1, 2, and 3 are computed using the optimal guidance law, suboptimal guidance law and proportion navigation law, respectively. The same is indicated in the following. Fig. 2 shows that k_1 increases as time-to-go decreases, and Fig. 3 shows that k_2 decreases as time-to-go decreases. As mentioned above, the coefficients of the suboptimal guidance law are the means of k_1 and k_2. In the first phase, the mean of k_1 is 2–3 times the original value, and in the final phase, it is 1/2–1/5 of the original value. Fig. 4 shows that the trajectory 2 is a good way to improve positioning accuracy because, for terminal guidance, the trajectory is steep, though needing more control energy.

Table 1
Comparison of three guidance laws.

Guidance law	Landing angle (degree)	Control energy consumption (substituted with the absolute value integral of over loading)	Miss distance (m)
Optimal guidance law	84.85	26.27	0.04
Suboptimal guidance law	80.74	30.10	0.07
Proportional navigation	38.94	12.47	0.28

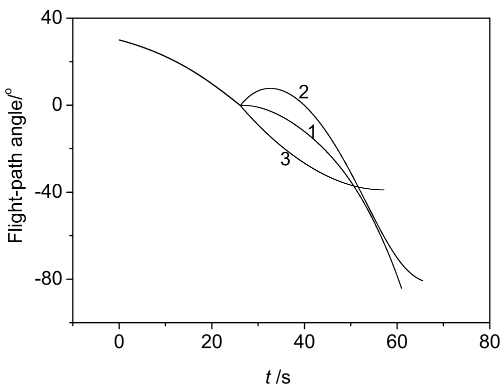

Fig. 5. Flight-path angle history.

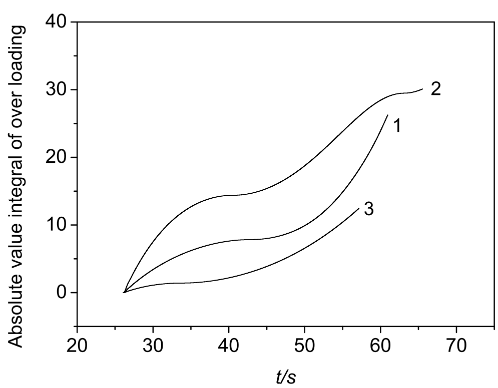

Fig. 6. Absolute value integral of overload.

The landing angles, control energy, and miss distances using the three different guidance laws are presented in Table 1. Here, the purpose is to compare control energy consumption focusing purely on the relative comparisons. Therefore, the absolute value integral of overload is substituted for the control energy consumption. The overload is defined as the ratio $\boldsymbol{N}/\boldsymbol{G}$, where $\boldsymbol{G}$ is the gravity acting on the guided projectile and $\boldsymbol{N}$ is the sum of all external forces, except gravity, acting on the guided projectile. The miss distance is defined as the straight-line distance between the landing point of the guided projectile and the target. Table 1 shows that the control energy consumption using the suboptimal guidance law is 15% higher than what is needed when using the optimal guidance law. Figs. 4–6 show that, when using the proportion navigation law, the control energy consumption is smallest; however, the landing angle is flattest and only 38.94°. Therefore, the proportion navigation law cannot satisfy the trajectory demand of GPS-guided munitions.

5. Conclusions

In order to optimize the combination of landing angle, miss distance, and control energy consumption, research regarding the optimal guidance law with multi-constraints is necessary. However, the optimal guidance law cannot generally be solved analytically when the motion equations include time-varying parameters. This paper attempts to solve this problem by combining the block pulse functions with the optimal control theory. Expanding the time-varying systems into the block pulse series, the system can be translated into a series of time-invariant systems, which can be solved with the optimal control theory. The results show that, using the proposed optimal guidance law, the landing angle is three times as large as what is obtained when using the proportion navigation law. The steep trajectory can increase strike accuracy, not only because it increases the GPS-based guidance accuracy, but also because it decreases the error caused by time step. The suboptimal guidance law has a very simple form and can save significantly computation resources; however it consumes more control energy.

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Appendix A

Definition of block pulse functions and their some elementary properties [16].

A block pulse function set $\phi_k(t)$ $(k = 1, 2, \ldots, m)$ can be defined in the interval $[0, T]$ as

$$\phi_k(t) = \begin{cases} 1, & (k-1)T/m \leqslant t < kT/m \\ 0, & \text{otherwise} \end{cases} \tag{35}$$

The block pulse functions, $\{\phi_0(t), \phi_1(t), \ldots, \phi_m(t)\}$, are disjoined from one another in $[0, T]$:

$$\phi_k(t)\phi_j(t) = \begin{cases} \phi_k(t), & k = j \\ 0, & k \neq j \end{cases} \tag{36}$$

The block pulse functions are orthogonal to one another in $[0, T]$:

$$\int_0^T \phi_k(t)\phi_j(t)\,\mathrm{d}t = \begin{cases} T/m, & k = j \\ 0, & k \neq j \end{cases} \tag{37}$$

An arbitrary integrable function $f(t)$ in $[0, T]$ can be approximately expanded into the block pulse series as

$$f(t) \cong \sum_{k=1}^{m} f_k \phi_k(t) \tag{38}$$

Let

$$\boldsymbol{\Phi}(t) = [\,\phi_1(t) \quad \phi_2(t) \quad \cdots \quad \phi_m(t)\,]^{\mathrm{T}} \tag{39}$$

The integral of the block pulse series can be written as follows:

$$\int_0^t \boldsymbol{\Phi}(t)\,\mathrm{d}t \cong \frac{T}{m}\boldsymbol{H}\boldsymbol{\Phi}(t), \quad 0 \leqslant t < T \tag{40}$$

where $\boldsymbol{H}$ is the $m \times m$ matrix:

$$\boldsymbol{H} = \frac{1}{2}\begin{bmatrix} 1 & 2 & 2 & \cdots & 2 \\ 0 & 1 & 2 & \cdots & 2 \\ 0 & 0 & 1 & \cdots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{41}$$

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